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Flavor Problem, Proton Decay And Neutrino Oscillations In SUSY Models With Anomalous $\mathcal{U}(1)$

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Abstract

We discuss how realistic supersymmetric models can be constructed by employing an anomalous $\mathcal{U}(1)$ flavor symmetry which also mediates supersymmetry breaking. A judicious choice of $\mathcal{U}(1)$ charges enables the first two squark families to be sufficiently heavy ($\gtrsim$ 10 TeV), so that flavor changing neutral currents as well as dimension five nucleon decay are adequately suppressed. Using the $SU(5)$ example, the charged fermion mass hierarchies, magnitudes of the CKM matrix elements, as well as the observed neutrino oscillations are simultaneously accommodated. We estimate the proton lifetime to be $\tau_p \sim 10^3 \cdot \tau_p$[minimal $SU(5)$], with the decay mode $p \to K\mu$ being comparable to $p \to K\nu_{\mu,\tau}$.

1 Introduction

Notwithstanding their enormous theoretical appeal, supersymmetric (SUSY) models provide several important challenges to the model builder. These include the problem of flavor changing neutral currents (FCNC), dimension five (including Planck scale suppressed) proton decay, and CP violating phases. Flavor non-conservation in SUSY theories is referred to as the supersymmetric flavor problem and is closely tied with the mechanism of SUSY breaking. New sources for FCNC in SUSY theories can arise from non-universal sparticle soft masses, and from a non-alignment (non-proportionality) of trilinear soft terms with the charged fermion Yukawa matrices. In $N = 1$ minimal supergravity (SUGRA) [1], universality and proportionality holds at the Planck scale ($M_P = 2.4 \cdot 10^{18}$ GeV). For estimating flavor changing processes one should renormalize the soft SUSY breaking terms between M_P and the SUSY breaking/electroweak scales. If a GUT scenario is considered one should also integrate out the heavy states which decouple at M_G, the GUT scale. These two procedures violate universality and proportionality [2], which could cause problems with FCNC. In gauge mediated SUSY breaking alignment holds at the low energy scale and FCNC are adequately suppressed.

An alternative approach for resolving the supersymmetric flavor problem is the so called decoupling solution [3, 7, 6], in which the FCNC are suppressed by large squark and slepton masses. In order to satisfy the existing experimental bounds [4] it is sufficient to have squarks (sleptons) in the mass range $\gtrsim$ 10 TeV. On the other hand, to avoid spoiling the gauge hierarchy, the stop mass (and also $\tilde{b}, \tilde{\tau}$ in case of large $\tan\beta$) should not exceed $\sim$ 1 TeV. The sparticles corresponding to the first two generations can be heavier, since their interactions with the higgs fields are suppressed by their Yukawa couplings. In the charged fermion sector we have, of course, the opposite hierarchical picture! A priori, without any symmetry reasons, it seems quite surprising to have a mass spectrum with such an inverse hierarchy.

In a recently proposed scenario [5], one possible mediator of SUSY breaking was assumed to be an anomalous $\mathcal{U}(1)$ symmetry, so that SUSY breaking mainly occurs through a non-zero D_A-term (of $\mathcal{U}(1)$), while the contributions from F-terms are relatively suppressed. Sparticles will gain soft masses if their $\mathcal{U}(1)$ charge is non zero. Otherwise, their soft mass will be relatively suppressed. It is tempting to exploit this $\mathcal{U}(1)$ also as a flavor symmetry. Since the top quark mass is close to the electroweak symmetry breaking scale, it is natural to assume that it arises through a renormalizable Yukawa coupling The $\mathcal{U}(1)$ charges of the higgs superfields are taken to be zero. Note that in the absence of additional symmetries the μ problem remains unresolved. The Yukawa couplings of the light families can be suppressed by prescribing them appropriate $\mathcal{U}(1)$ charges. It follows that sparticles corresponding to the light fermions will have large soft masses in comparison to their counterparts from the third family. If the contribution to the soft masses from

D_A-term is dominant and in the 10 TeV range, the supersymmetric flavor problem will be resolved.

In this paper we attempt to develop this approach within the framework of $SU(5)$ GUT and study some of its phenomenological implications (for earlier related works see [6]-[8]). Employing an anomalous $\mathcal{U}(1)$ as a mediator of SUSY breaking, and as a flavor symmetry, we obtain a suitable mass spectrum for proper suppression of FCNC. It turns out that this also leads to a strong (and desirable) suppression of the dominant nucleon decay in minimal SUSY $SU(5)$, since in the internal loops of the $d = 6$ nucleon decay diagrams, there appear sparticles belonging to the first two families. In our scenario the dominant decays occur through diagrams in which sparticles of the third generation participate, and for adequate suppression the regime with intermediate or low $\tan\beta$ is required. It is worth stressing that the neutrino and charged lepton decay channels are comparable in magnitude, with the proton lifetime estimated to be $\tau_p \sim 10^3 \tau_0$ (where $\tau_0 \sim 10^{29\pm2}$ yr. is the proton lifetime in minimal SUSY $SU(5)$, assuming squark and gaugino masses around 1 TeV). Due to $\mathcal{U}(1)$ flavor symmetry, all Planck scale mediated $d = 5$ baryon number violating operators are also adequately suppressed. The model is also compatible with the various neutrino oscillation scenarios that are in agreement with the atmospheric and solar neutrino data [10, 11]. We stress bi-maximal vacuum neutrino mixing scenario, with the $\mathcal{U}(1)$ symmetry once again playing a crucial role [12, 13, 14]. We also indicate how the large and small mixing angle MSW oscillations for resolving the solar neutrino anomaly can be realized.

The paper is organized as follows: in Section 2 we discuss SUSY breaking through an anomalous $\mathcal{U}(1)$ symmetry, and show how the desirable sparticle spectrum needed for suppression of FCNC can be obtained. Some necessary conditions which should be satisfied are pointed out. We also discuss suppression of nucleon decay and present the appropriate suppression factors which do not depend on GUT physics, but are closely tied to the low energy sector. In Section 3 we present an $SU(5)$ example in which (the same) anomalous $\mathcal{U}(1)$ symmetry is exploited as a flavor symmetry to provide a natural understanding of hierarchies between charged fermion masses and their mixings. We briefly explain how the bad asymptotic $SU(5)$ mass relations $\hat{M}_d^0 = \hat{M}_e^0$ involving the light families are avoided in our approach. We discuss the various neutrino oscillation scenarios which simultaneously accommodate the atmospheric and solar neutrino puzzles. Estimates for the nucleon decay widths are also presented. Our conclusions are summarized in Section 4.

2 SUSY breaking anomalous $\mathcal{U}(1)$, FCNC and nucleon decay

We employ the proposal of ref. [5] and consider an anomalous $\mathcal{U}(1)$ symmetry as a mediator of SUSY breaking. It is well known that anomalous $\mathcal{U}(1)$ symmetries often emerges from strings. The cancellation of its anomalies occur through the Green-Schwarz mechanism [15], and the associated Fayet-Iliopoulos term is given by [16]

$$\xi \int d^4\theta V_A \ , \qquad \xi = \frac{g_A^2 M_P^2}{192\pi^2}\mathrm{Tr}Q \ . \tag{1}$$

The D_A-term is

$$\frac{g_A^2}{8}D_A^2 = \frac{g_A^2}{8}\left(\Sigma Q_a|\varphi_a|^2 + \xi\right)^2 \ , \tag{2}$$

where Q_a is the 'anomalous' charge of φ_a superfield.

Let us introduce a singlet superfield X with $\mathcal{U}(1)$ charge $Q_X = -1$. Assuming $\mathrm{Tr}Q > 0$ ($\xi > 0$), the cancellation of (2) fixes the VEV of the scalar component of X:

$$\langle X \rangle = \sqrt{\xi} \ , \tag{3}$$

with SUSY unbroken at this stage. Including a mass term for X in the superpotential,

$$W_m = \frac{m}{2}X^2 \ , \tag{4}$$

the cancellation of D_A will be partial, and SUSY will be broken due to non-zero F and D terms. Taking into account (2) and (4), the potential for X will have the form

$$V = m^2|X|^2 + \frac{g_A^2}{8}\left(\xi - |X|^2\right)^2 \ . \tag{5}$$

Minimization of (5) gives

$$X^2 = \xi - \frac{4m^2}{g_A^2} \ , \tag{6}$$

along which

$$\langle D_A \rangle = \frac{4m^2}{g_A^2} \ , \qquad \langle F_X \rangle \simeq m\sqrt{\xi} \ . \tag{7}$$

From (2), taking into account (6), (7), for the soft scalar masses squared (mass^2) we have

$$m_{\tilde{\varphi}_a}^2 = Q_a m^2 \ . \tag{8}$$

Thus, the scalar components of superfields which have non-zero $\mathcal{U}(1)$ charges gain masses through $\langle D_A \rangle$.

We will assume that the VEV of X is somewhat below M_P, namely

$$\frac{\langle X \rangle}{M_P} \equiv \epsilon \simeq 0.22 \ , \tag{9}$$

while the scale m is in the range ~ 10 TeV (see below). Those states which have zero $\mathcal{U}(1)$ charges will gain soft masses of the order of gravitino mass $m_{3/2}$ from the Kähler potential

$$m_{3/2} = \frac{F_X}{\sqrt{3} M_P} = m \frac{\epsilon}{\sqrt{3}} \ , \tag{10}$$

which, for $m = 10$ TeV, is relatively suppressed (~ 1 TeV). The gaugino masses also will have the same magnitudes

$$M_{\tilde{G}_i} \sim m_{3/2} \sim 1 \ \text{TeV} \ . \tag{11}$$

The mass term (4) violates the $\mathcal{U}(1)$ symmetry and is taken to be in the 10 TeV range. Its origin may lie in a strong dynamics where m is replaced by the VEV of some superfield(s) [5, 17]. One possibility is to introduce a singlet superfield Z with $Q_Z = 2$, and vector-like superfields $\bar{Q} + Q$ ($Q_{\bar{Q}} = Q_Q = 0$), assumed to be a doublet-antidoublet pair of a strong $SU(2)$ gauge group. Then, imposing an additional global symmetry,

$$Z \to e^{\mathrm{i}\alpha} Z \ , \quad \bar{Q}Q \to e^{-\mathrm{i}\alpha} \bar{Q}Q \ , \tag{12}$$

the lowest term in the superpotential will be

$$W_0 = \lambda \frac{\bar{Q}Q}{M_P^2} Z X^2 \ . \tag{13}$$

Assuming that $SU(2)$ becomes strong at scale Λ, the non-perturbative superpotential induced by the instantons will have the form [18]

$$W_{\text{inst}} = \frac{\Lambda^5}{\bar{Q}Q} \ , \tag{14}$$

and the scalar superpotential will be [1]:

$$W_s = \lambda \frac{\bar{Q}Q}{M_P^2} Z X^2 + \frac{\Lambda^5}{\bar{Q}Q} \ . \tag{15}$$

[1] The non-perturbative term (14) violates global symmetry in (12). This can happen if the symmetry is anomalous.

The potential built from the F and D-terms has the form

$$V_s = \Sigma |F_a|^2 + \frac{g_A^2}{8} D_A^2 \ , \tag{16}$$

where

$$F_a = \frac{dW_s}{d\varphi_a} \ , \qquad D_A = \xi - |X|^2 + 2|Z|^2 \ . \tag{17}$$

It is easy to verify that there is no solution along which the F and D-terms simultaneously vanish. Minimization of (16) gives the following solutions

$$X^2 = \frac{4}{3}\xi - \frac{16}{3}\frac{m^2}{g_A^2} \ , \qquad Z^2 = \frac{1}{6}\xi - \frac{2}{3}\frac{m^2}{g_A^2} \ ,$$
$$\bar{Q}^4 = Q^4 = \frac{9m^2 M_P^2}{2\lambda}\left(1 - \frac{9\sqrt{3}}{8}\frac{mM_P^2}{\xi\sqrt{\xi}}\right) \ , \tag{18}$$

where

$$m^2 = \frac{\sqrt{6}\Lambda^5}{\xi\sqrt{\xi}} \ . \tag{19}$$

From (9), (18) we find

$$\epsilon_Z \equiv \frac{\langle Z\rangle}{M_P} = \frac{1}{2\sqrt{2}}\epsilon \simeq 0.08 \ , \qquad \sqrt{\xi} = \frac{\sqrt{3}}{2} M_P \epsilon \ . \tag{20}$$

Substituting (18) in (17), we readily obtain the expression for $\langle D_A\rangle$ given in (7), and expression (8) (for calculating soft masses) is valid. Assuming that $\Lambda \simeq 3.3 \cdot 10^{12}$ GeV, from (19) we obtain the desirable value for $m(\simeq 10$ TeV). In this example, among the non-zero F-terms, it is F_Z which dominates and provides the dominant contribution to the gravitino mass (~ 1 TeV) in (10).

Turning now to the question of FCNC, we require that the 'light' quark-lepton superfields carry non-zero $\mathcal{U}(1)$ charges. This means that the soft masses of their scalar components are in the 10 TeV range, which automatically suppresses flavor changing processes such as $K^0 - \bar{K}^0$, $\mu \to e\gamma$ etc., thereby satisfying the present experimental bounds [4]. To prevent upsetting the gauge hierarchy, the third generation up squarks must have masses no larger than a TeV or so [8] (hence they have zero $\mathcal{U}(1)$ charge). The same applies to sbottom and stau for large $\tan\beta$ since, for $\lambda_b \sim \lambda_\tau \sim 1$, large masses ($\gtrsim 10$ TeV) of $\tilde{b}$ and $\tilde{\tau}$ would spoil the gauge hierarchy.

Although the tree level mass of the stop can be arranged to be in the 1 TeV range by the $\mathcal{U}(1)$ symmetry, the 2-loop contributions from heavy sparticles of the first two generations can drive the stop mass^2 negative [8]. This is clearly unacceptable, and one proposal for avoiding it [6] requires the existence of new states in the multi-TeV range. The dangerous contribution which comes from 2-loop diagrams is proportional to

$$\Sigma m^2_{\tilde{\varphi}_a} T_a \ , \tag{21}$$

where $m^2_{\tilde{\varphi}_a}$ (see (8)) is the soft mass^2 of $\tilde{\varphi}_a$, and T_a is the Dynkin index of the appropriate representation. The representations and $\mathcal{U}(1)$ charges of the new states should be chosen so that (21) vanishes, namely

$$\Sigma Q_a T_a = 0 \ . \tag{22}$$

We will later see how this is implemented in the $SU(5)$ example.

Let us now turn to some implications for proton decay. We assume that $d = 5$ baryon number violating operators arise from the couplings

$$qAqT + qBl\bar{T} \ , \tag{23}$$

after integration of color triplets $T, \bar{T}$ with mass $M_T \sim 2 \cdot 10^{16}$ GeV (we consider triplet couplings with left-handed matter, which provide the dominant contribution to nucleon decay). After wino dressing of appropriate $d = 5$ operators, the resulting $d = 6$ operators causing proton to decay into the neutrino and charged lepton channels have the respective forms [19, 20]:

$$\frac{g_2^2}{M_T} \alpha (u_a d_b^i)(d_c^j \nu^k) \varepsilon^{abc} \ , \tag{24}$$

$$\frac{g_2^2}{M_T} \alpha' (u_a d_b^i)(u_c e^j) \varepsilon^{abc} \ , \tag{25}$$

where

$$\alpha = -[(L_d^+ \hat{B} L_e)_{jk} (L_u^+ \hat{A} L_d^*)_{mn} + (L_d^+ \hat{A} L_u^*)_{jm} (L_d^+ \hat{B} L_e)_{nk}] V_{mi} (V^+)_{n1} I(\tilde{u}^m, \tilde{d^n}) +$$
$$[(L_u^+ \hat{A} L_d^*)_{1i} (L_u^+ \hat{B} L_e)_{mk} - (L_d^+ \hat{A} L_u^*)_{im} (L_u^+ \hat{B} L_e)_{ik}] V_{mj} I(\tilde{u^m}, \tilde{e^k}) \ , \tag{26}$$

$$\alpha' = [-(L_u^+ \hat{A} L_d^*)_{1i} (L_d^+ \hat{B} L_e)_{mj} + (L_u^+ \hat{A} L_d^*)_{1m} (L_d^+ \hat{B} L_e)_{ij}] (V^+)_{m1} I(\tilde{d^m}, \tilde{\nu^j}) +$$
$$[(L_u^+ \hat{B} L_e)_{1j} (L_u^+ \hat{A} L_d^*)_{mn} + (L_u^+ \hat{A} L_d^*)_{1m} (L_e^T \hat{B}^T L_u^*)_{jn}] (V^+)_{m1} V_{ni} I(\tilde{d^m}, \tilde{u}^n) \ . \tag{27}$$

L's are unitary matrices which rotate the left handed fermion states to diagonalize the mass matrices, and I's are functions obtained after loop integration and depend on the SUSY particle masses circulating inside the loop. For example [19],

$$I(\tilde{u},\tilde{d}) = \frac{1}{16\pi^2}\frac{m_{\tilde{W}}}{m_{\tilde{u}}^2 - m_{\tilde{d}}^2}\left(\frac{m_{\tilde{u}}^2}{m_{\tilde{u}}^2 - m_{\tilde{W}}^2}\ln\frac{m_{\tilde{u}}^2}{m_{\tilde{W}}^2} - \frac{m_{\tilde{d}}^2}{m_{\tilde{d}}^2 - m_{\tilde{W}}^2}\ln\frac{m_{\tilde{d}}^2}{m_{\tilde{W}}^2}\right) , \tag{28}$$

with similar expressions for $I(\tilde{d},\tilde{\nu})$ and $I(\tilde{u},\tilde{e})$.

Consider those diagrams in which sparticles of the first two families participate. Since their masses are large ($\gtrsim$ 10 TeV) compared to the case with minimal $N = 1$ SUGRA, we expect considerably suppression of proton decay. For minimal $N = 1$ SUGRA, $m_{\tilde{u}} \sim m_{\tilde{d}} \sim m_{\tilde{W}} \sim m_{3/2} \sim 1$ TeV, and (28) can be approximated by

$$I_0 \approx \frac{1}{16\pi^2}\frac{1}{m_{3/2}} . \tag{29}$$

In the $\mathcal{U}(1)$ mediated SUSY breaking scenario, expression (28) takes the form

$$I' \approx \frac{1}{16\pi^2}\frac{m_{\tilde{W}}}{m_{\tilde{q}}^2} \equiv \eta I_0 \tag{30}$$

The nucleon lifetime in this case will be enhanced by the factor $\frac{1}{\eta^2} \sim 10^4$.

Of course, there exist diagrams in which one sparticle from the third and one from the 'light' families participate. In this case, (28) takes the form

$$I'' \approx \frac{1}{16\pi^2}\frac{2m_{\tilde{W}}}{m_{\tilde{q}}^2}\ln\frac{m_{\tilde{q}}}{m_{\tilde{W}}} \equiv \eta' I_0 \tag{31}$$

and the corresponding proton lifetime will be $\sim \frac{1}{\eta'^2} \sim 500$ times large.

As pointed out in [20] (within minimal $N = 1$ SUGRA), the contribution from diagrams in which sparticles from the third generation participate could be comparable with those arising from the light generation sparticle exchange. Since minimal SUSY $SU(5)$ gives unacceptably fast proton decay with $\tau_0 \sim 10^{29\pm2}$ yr, care must exercised in realistic model building (the situation is exacerbated if $\tan\beta$ is large). This problem is easily avoided in the anomalous $\mathcal{U}(1)$ mediated SUSY breaking scenario. Note that since the dominant contribution comes from the second term of (26), and in the internal lines of the appropriate nucleon decay diagram there necessarily runs one slepton state, even if the latter belongs to third family, it can have mass in the 10 TeV range if $\tan\beta$ is either of intermediate ($\sim 10 - 15$) or low value (this is required for preserving the desired gauge hierarchy). Thus, thanks to the anomalous $\mathcal{U}(1)$ symmetry, in addition to avoiding dangerous FCNC, one can also obtain adequate suppression of nucleon decay. Interestingly, this disfavors the large $\tan\beta$ regime which could be a characteristic feature of this class of models!

3 An $SU(5)$ example

Let us now consider in detail a SUSY $SU(5)$ GUT and show how things discussed in the previous section work out in practice.

3.1 Charged fermion masses and mixings

We exploit the anomalous $\mathcal{U}(1)$ as a flavor symmetry [9] to help provide a natural understanding of the hierarchies between the charged fermion masses and their mixings. In these considerations the parameter $\epsilon \simeq 0.22$ (see (9)) plays an essential role. The three families of matter in $(10+\bar{5})$ representations, and higgs superfields $\bar{H}(\bar{5})+H(5)$ [2] have the following transformation properties under $\mathcal{U}(1)$:

$$
\begin{gathered}
Q_{10_3}=0 \ , \quad Q_{10_2}=2 \ , \quad Q_{10_1}=3 \\
Q_{\bar{5}_1}=2+n \ , \quad Q_{\bar{5}_2}=Q_{\bar{5}_3}=n \ , \quad Q_{\bar{H}}=Q_H=0 \ .
\end{gathered}
\tag{32}
$$

The couplings relevant for the generation of up type quark masses are given by

$$
\begin{array}{c c}
 & \begin{array}{ccc} 10_1 & 10_2 & 10_3 \end{array} \\
\begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} & \left(\begin{array}{ccc} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{array}\right) H \ ,
\end{array}
\tag{33}
$$

while those responsible for down quark and charged lepton masses are

$$
\begin{array}{c c}
 & \begin{array}{ccc} \bar{5}_1 & \bar{5}_2 & \bar{5}_3 \end{array} \\
\begin{array}{c} 10_1 \\ 10_2 \\ 10_3 \end{array} & \left(\begin{array}{ccc} \epsilon^5 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & 1 & 1 \end{array}\right) \epsilon^n \bar{H} \ .
\end{array}
\tag{34}
$$

Upon diagonalization of (33), (34) we obtain

$$
\lambda_t \sim 1 \ , \quad \lambda_u : \lambda_c : \lambda_t \sim \epsilon^6 : \epsilon^4 : 1 \ . \tag{35}
$$

$$
\begin{gathered}
\lambda_b \sim \epsilon^n \ , \quad \lambda_d : \lambda_s : \lambda_b \sim \epsilon^5 : \epsilon^2 : 1 \ , \\
\lambda_\tau \sim \epsilon^n \ , \quad \lambda_e : \lambda_\mu : \lambda_\tau \sim \epsilon^5 : \epsilon^2 : 1 \ ,
\end{gathered}
\tag{36}
$$

where $n=0,1,2$ determines the value of $\tan\beta$,

[2] We assume the presence of Z_2 matter parity which distinguishes the matter and higgs superfields and prevents rapid proton decay.

$$\tan\beta \sim \epsilon^n \frac{m_t}{m_b} \ . \tag{37}$$

From (33) and (34), we obtain

$$V_{us} \sim \epsilon \ , \ V_{cb} \sim \epsilon^2 \ , \ V_{ub} \sim \epsilon^3 \ . \tag{38}$$

We see that the $\mathcal{U}(1)$ symmetry yields desirable hierarchies (35), (36) of charged fermion Yukawa couplings as well as the magnitudes of the CKM matrix elements (38).

The reader will note, however, that (34) implies the asymptotic mass relations $\hat{M}_d^0 = \hat{M}_e^0$, which are unacceptable for the two light families. This is readily avoided through the mechanism suggested in [13] by employing two pairs of $(\overline{15}+15)_{1,2}$ matter states. Namely, with $\mathcal{U}(1)$ charges

$$Q_{15_1} = -Q_{\overline{15}_1} = 3 \ , \qquad Q_{15_2} = -Q_{\overline{15}_2} = 2 \ , \tag{39}$$

consider the couplings

$$\begin{array}{c} \\ \overline{15}_1 \\ \overline{15}_2 \end{array} \!\!\begin{array}{c} \begin{array}{ccc} 10_1 & 10_2 & 10_3 \end{array} \\ \left(\begin{array}{ccc} 1 & 0 & 0 \\ \epsilon & 1 & 0 \end{array}\right) \end{array} \Sigma \ , \quad \begin{array}{c} \\ \overline{15}_1 \\ \overline{15}_2 \end{array} \!\!\begin{array}{c} \begin{array}{cc} 15_1 & 15_2 \end{array} \\ \left(\begin{array}{cc} 1 & 0 \\ \epsilon & 1 \end{array}\right) \end{array} M_{15} \ , \tag{40}$$

where Σ is the scalar 24-plet whose VEV breaks $SU(5)$ down to $SU(3)_c \times SU(2)_L \times U(1)_Y$. For $M_{15} \sim \langle\Sigma\rangle$, we see that the 'light' $q_{1,2}$ states reside both in $10_{1,2}$ and $15_{1,2}$ states with similar 'weights'. At the same time, the other light states from 10-plets (u^c and e^c) will not be affected because the 15-plets do not contain fragments with the relevant quantum numbers. Thus, the relations $m_s^0 = m_\mu^0$ and $m_d^0 = m_e^0$ are avoided, while $m_b^0 = m_\tau^0$ still holds since the terms in (40) do not affect 10_3.

As far as the sparticle spectrum is concerned, since the superfields $10_3, \bar{H}, H$ have zero $\mathcal{U}(1)$ charges, the soft masses of their scalar components will be in the 1 TeV range,

$$m_{\tilde{10}_3} \sim m_{\bar{H}} \sim m_H \sim m_{3/2} = 1 \text{ TeV} \ , \tag{41}$$

while for $10_{1,2}$ and $\bar{5}_1$ we have

$$m_{\tilde{10}_1} \sim m_{\tilde{10}_2} \sim m_{\tilde{\bar{5}}_1} \sim m \sim 10 \text{ TeV} \ . \tag{42}$$

The soft masses of the scalar fragments from $\bar{5}_{2,3}$ depend on the value of n, and for $n \neq 0$, they also will be in the 10 TeV range, which is preferred for proton stability.

In order to satisfy condition (22) and avoid color instability in our model, we will introduce one pair of $\bar{F}(\bar{5}) + F(5)$ supermultiplets with $\mathcal{U}(1)$ charges

$$Q_{\bar{F}} = Q_F = -\frac{1}{2}(17+3n) \ , \tag{43}$$

and with the following transformation properties under the symmetry in (12),

$$\bar{F}F \to e^{-(10+n)\mathrm{i}\alpha}\bar{F}F \ . \tag{44}$$

The superpotential coupling which generates mass term for these states is given by

$$W_F = M_P\left(\frac{Z}{M_P}\right)^{10+n}\left(\frac{X}{M_P}\right)^{3-n}\bar{F}F \ , \tag{45}$$

from which, after substituting the VEVs (9) and (20), we obtain

$$M_F = M_P\epsilon_Z^{10+n}\epsilon^{3-n} = \begin{cases} 200 \text{ TeV} & \text{if } n=0 \\ 100 \text{ TeV} & \text{if } n=1 \\ 40 \text{ TeV} & \text{if } n=2 \end{cases} \tag{46}$$

Therefore, the masses of these additional states are considerable more than a TeV range. It is easy to verify that M_F^2 dominates over the negative soft mass^2 ($= -\frac{17+3n}{2}m^2 \simeq -(30 \text{ TeV})^2$) for all possible values of $n(=0,1,2)$, so that color will be unbroken. Furthermore, taking into account (32) and (43), it is easily checked that the condition (22) which prevents the stop quark mass^2 from becoming negative is automatically satisfied.

Finally, since the $\mathcal{U}(1)$ charges of $15_{1,2}$ states are the same as those of $10_{1,2}$'s, the soft mass^2 terms for light $\tilde{q}_{1,2}$ fragments are unchanged so that (22), with the choice of charges in (43), still holds.

3.2 Neutrino oscillations

We next demonstrate how the solar and atmospheric neutrino data can be accommodated within the $SU(5)$ scheme. We stress the bi-maximal vacuum oscillation scenario, but also point out how the small (or large) mixing angle MSW oscillations can be realized. Indeed, the picture is similar to our previously considered $SU(5)$ [13] and $SO(10)$ [14] scenarios.

Since the states $\bar{5}_2$ and $\bar{5}_3$ have the same $\mathcal{U}(1)$ charge (see (32)), we can expect naturally large $\nu_\mu - \nu_\tau$ mixing. This also can be seen from the texture in (34). Introducing an $SU(5)$ singlet right handed neutrino $\mathcal{N}_3$ with suitable mass, the state 'ν_3' can acquire the mass relevant for the atmospheric neutrino puzzle. At this stage the other two neutrino states are massless.

Large $\nu_e - \nu_{\mu,\tau}$ mixing can be obtained by invoking the mechanism suggested in [12] which naturally yields 'maximal' mixings between neutrino flavors. For this we need two additional $SU(5)$ singlet states $\mathcal{N}_1$, $\mathcal{N}_2$. Under $\mathcal{U}(1)$, the $\mathcal{N}_i$ states carry charges:

$$Q_{\mathcal{N}_1} = -Q_{\mathcal{N}_2} = n+2 \ , \qquad Q_{\mathcal{N}_3} = 0 \ . \tag{47}$$

The relevant couplings are

$$W_{\mathcal{N}_3} = M_{\mathcal{N}_3}\mathcal{N}_3^2 + \epsilon^n (a\epsilon^2 \bar{5}_1 + b\bar{5}_2 + c\bar{5}_3) H \mathcal{N}_3, \tag{48}$$

$$\begin{array}{c} \\ \bar{5}_1 \\ \bar{5}_2 \\ \bar{5}_3 \end{array} \!\!\begin{array}{c} \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{pmatrix} \epsilon^{2n+4} & 1 \\ \epsilon^{2n+2} & 0 \\ \epsilon^{2n+2} & 0 \end{pmatrix} \end{array} H \ , \qquad \begin{array}{c} \\ \mathcal{N}_1 \\ \mathcal{N}_2 \end{array} \!\!\begin{array}{c} \begin{array}{cc} \mathcal{N}_1 & \mathcal{N}_2 \end{array} \\ \begin{pmatrix} \epsilon^{2n+4} & 1 \\ 1 & 0 \end{pmatrix} \end{array} M_{\mathcal{N}} \ , \tag{49}$$

where a, b, c are dimensionless coefficients. Note that there also exists the coupling $M'\epsilon^{2+n}\mathcal{N}_1\mathcal{N}_3$ which, if properly suppressed (see below), will not be relevant.

Let us choose the basis in which the charged lepton matrix (34) is diagonal. This choice is convenient because the matrix which diagonalizes the neutrino mass matrix will then coincide with the lepton mixing matrix. The hierarchical structure of the couplings in (48) will not be altered, while the 'Dirac' and 'Majorana' masses from (49) will respectively have the forms

$$m_D = \begin{pmatrix} \epsilon^{2n+4} & 1 \\ \epsilon^{2n+2} & \epsilon^2 \\ \epsilon^{2n+2} & \epsilon^2 \end{pmatrix} h_u \ , \qquad M_R = \begin{pmatrix} \epsilon^4 & 1 \\ 1 & 0 \end{pmatrix} M_{\mathcal{N}} \ . \tag{50}$$

Taking

$$M' \ll M_{\mathcal{N}_3}/\epsilon^{2n} \ , \qquad M_{\mathcal{N}} \gtrsim \frac{M'^2\epsilon^{2n}}{M_{\mathcal{N}_3}} \tag{51}$$

and the other coefficients of order unity, integration of the $\mathcal{N}$ states leads to the following 'light' neutrino mass matrix:

$$\hat{m}_\nu = \hat{A}m + \hat{B}m' \ , \tag{52}$$

where

$$m \equiv \frac{\epsilon^{2n}h_u^2}{M_{\mathcal{N}_3}} \ , \qquad m' \equiv \frac{\epsilon^{2n+2}h_u^2}{M_{\mathcal{N}}} \ , \tag{53}$$

$$\hat{A} = \begin{pmatrix} a^2\epsilon^4 & ab\epsilon^2 & ac\epsilon^2 \\ ab\epsilon^2 & b^2 & bc \\ ac\epsilon^2 & bc & c^2 \end{pmatrix} m \ ,$$

$$\hat{B} = \begin{pmatrix} \epsilon^2 & 1 & 1 \\ 1 & \epsilon^2 & \epsilon^2 \\ 1 & \epsilon^2 & \epsilon^2 \end{pmatrix} m' \ \cdot \tag{54}$$

For

$$M_{\mathcal{N}_3} \simeq \epsilon^{2n} \cdot 10^{15}\ \mathrm{GeV}\ , \qquad M_{\mathcal{N}} \simeq \epsilon^{2n+2} \cdot 10^{18}\ \mathrm{GeV}\ , \tag{55}$$

the ‘light’ eigenvalues are

$$\begin{aligned} m_{\nu_3} &\simeq m(b^2 + c^2 + a^2\epsilon^4) \sim 3 \cdot 10^{-2}\ \mathrm{eV}\ , \\ m_{\nu_1} &\simeq m_{\nu_2} \simeq m' \sim 3 \cdot 10^{-5}\ \mathrm{eV}\ . \end{aligned} \tag{56}$$

Ignoring CP violation the neutrino mass matrix (52) can be diagonalized by the orthogonal transformation $\nu_\alpha = U_\nu^{\alpha i}\nu_i$, where $\alpha = e, \mu, \tau$ denotes flavor indices, $i = 1, 2, 3$ the mass eigenstates, and U_ν takes the form

$$U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & s_1 \\ -\frac{1}{\sqrt{2}}c_\theta & \frac{1}{\sqrt{2}}c_\theta & s_\theta \\ \frac{1}{\sqrt{2}}s_\theta & -\frac{1}{\sqrt{2}}s_\theta & c_\theta \end{pmatrix}\ , \tag{57}$$

with

$$\tan\theta = \frac{b}{c}\ , \qquad s_1 = \frac{a\epsilon^2}{\sqrt{b^2 + c^2}}\ , \tag{58}$$

and $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$. From (52)-(58) the solar and atmospheric neutrino oscillation parameters are

$$\begin{aligned} \Delta m_{21}^2 &\sim 2m'^2\epsilon^2 \simeq 10^{-10}\ \mathrm{eV}^2\ , \\ \mathcal{A}(\nu_e \to \nu_{\mu,\tau}) &= 1 - \mathcal{O}(\epsilon^4)\ , \end{aligned} \tag{59}$$

$$\begin{aligned} \Delta m_{32}^2 &\simeq m_{\nu_3}^2 \sim 10^{-3}\ \mathrm{eV}^2\ , \\ \mathcal{A}(\nu_\mu \to \nu_\tau) &= \frac{4b^2c^2}{(b^2 + c^2)^2} - \mathcal{O}(\epsilon^4)\ , \end{aligned} \tag{60}$$

where the oscillation amplitudes are defined as

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = 4\Sigma_{j<k} U_\nu^{\alpha j} U_\nu^{\alpha k} U_\nu^{\beta j} U_\nu^{\beta k}\ . \tag{61}$$

We see that the solar neutrino puzzle is explained by maximal vacuum oscillations of ν_e into $\nu_{\mu,\tau}$. For $b \sim c$ the $\nu_\mu - \nu_\tau$ mixing is naturally large, as suggested by the atmospheric anomaly. For $b \simeq c$ the $\nu_\mu - \nu_\tau$ mixing will be even maximal, and ν_e oscillations will be 50% into ν_μ and 50% into ν_τ.

As far as the small angle MSW solution for the solar neutrino puzzle is concerned, from (34) we see that the expected mixing between ν_e and $\nu_{\mu,\tau}$ states is $\sim \epsilon^2$, which provides the desirable value $\sin^2 2\theta \sim 4\epsilon^4 \simeq 10^{-2}$. To obtain $\nu_e - \nu_{\mu,\tau}$ oscillations, we can introduce a $SU(5)$ singlet state N (instead of $\mathcal{N}_{1,2}$ states), which will provide mass in the

10^{-3} eV range to the 'ν_2' state, so that the small angle MSW oscillation for explaining the solar neutrino deficit is realized.

Large mixing angle MSW solution is obtained by keeping the $\mathcal{N}_{1,2}$ states with the transformation properties in (47). Maximal $\nu_e - \nu_{\mu,\tau}$ oscillations will still hold, and the desired scale ($\sim 10^{-6}$ eV^2) can be generated by taking $M_{\mathcal{N}} \simeq \epsilon^{2n+2} \cdot 10^{16}$ GeV in (53). The oscillation picture (60) for the atmospheric neutrinos will be unchanged.

3.3 Nucleon decay in $SU(5)$

Turning to the issue of nucleon decay in $SU(5)$, we will take $n \neq 0$ in (32), which provides soft masses for $\bar{5}_{2,3}$ states in the 10 TeV range. As pointed out in section 2, this will enhance proton stability. For decays with neutrino emission, in the relevant diagrams there circulate $\tilde{t}$ and $\tilde{\mu}(\tilde{\tau})$. Using the forms of (33), (34), and taking into account (26), (30), one estimates from (24),

$$\tau(p \to K\nu_{\mu,\tau}) \sim \frac{1}{\eta'^2}\left(\frac{\sin^2\theta_c}{V_{ts}}\right)^2 \tau_0 \sim 2 \cdot 10^3 \tau_0 \ , \tag{62}$$

where θ_c is the Cabibbo angle and τ_0 is the proton lifetime in minimal $SU(5)$ model combined with SUGRA [in obtaining (62), we took into account that $\tau_0 \propto (\lambda_s \lambda_c \sin^2\theta)^2$].

As far as decays with emission of charged leptons are concerned, there are diagrams inside which circulate $\tilde{t}$, $\tilde{b}$ states, which are in the 1 TeV range. It turns out that these diagrams provide the dominant contribution to proton decay. However, considerable suppression relative to minimal $SU(5)$ still occurs due to the small mixings between the third and light generations, and also because the baryon-meson-charged lepton matrix element is relatively suppressed [19]. From all this, taking into account (25), (27), (33), (34), for the dominant decay we find

$$\tau(p \to K\mu) \sim 10 \left(\frac{\sin^2\theta_c}{V_{ub}}\right)^2 \tau_0 \sim 10^3 \tau_0 \ . \tag{63}$$

In summary, the color triplet mediated proton decay modes are adequately suppressed and interestingly, the decays into the charged lepton and neutrino channels are comparable.

Before concluding, let us note that the Planck scale suppressed baryon number violating $d = 5$ operator $\frac{1}{M_P} q_1 q_1 q_2 l_{2,3}$, which could cause unacceptably fast proton decay, is also suppressed, since it emerges from the coupling

$$\frac{1}{M_P}\left(\frac{X}{M_P}\right)^{8+n} 10_1 10_1 10_2 \bar{5}_{2,3} \ , \tag{64}$$

with the suppression guaranteed by the $\mathcal{U}(1)$ symmetry.

4 Conclusions

In this paper we have discussed SUSY models which are accompanied by an anomalous $\mathcal{U}(1)$ symmetry. If the latter mediates SUSY breaking, crucial suppression of FCNC as well as dimension five proton decay can be achieved. If the same $\mathcal{U}(1)$ also acts as flavor symmetry, one can provide a natural qualitative explanation of the hierarchies between the charged fermion masses and the values of CKM matrix elements.

An example based on $SU(5)$ is worked out in detail, with neutrino oscillations also taken into account. The $\mathcal{U}(1)$ flavor symmetry also adequately suppresses the Planck scale induced baryon number violating $d = 5$ operators. The mechanisms discussed in this paper can be extended to a variety of GUTs such as $SO(10)$ and $SU(5+N)$.

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